

**Aero-structural design optimization of long-span bridges:  
From linear to nonlinear aeroelasticity-driven perspectives**  
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## ABSTRACT

Structural optimization techniques have been demonstrated to be a powerful tool for the cost-effective design of bridges under aeroelastic considerations, particularly when applied to super-long span suspension or cable-stayed bridges. The efficacy of this methodology relies on the comprehensive and accurate formulation of the wind-resistant design problem. The analysis of the wind-induced responses, such as flutter and buffeting, has been typically addressed in the industry by adopting multi-mode analysis techniques using linear force modeling approaches based on the fundamental contributions of Prof. Davenport and Prof. Scanlan. In the same way, the aero-structural optimization frameworks previously developed by the authors have followed this approach by mimicking the design goals and specifications of real bridge projects in the formulation of the optimization problem. However, wind tunnel tests and on-site monitoring measurements have shown that under some circumstances, the so-called linear aeroelasticity models fail in predicting the bridge responses. Hence, several nonlinear aeroelastic methods have been developed in the last decades, including the corrected quasi-steady theory (QST) model, band superposition model, hybrid nonlinear model, rheological model, artificial neural networks (ANN) based model, and Volterra models, among others. These methods should be advanced in order to define deck shape-dependent accurate models that permit their implementation into design optimization frameworks to achieve cost-effective and safe bridge design. This study reviews the effectiveness of these methods and discusses practical directions to follow to adequately implement nonlinear aeroelasticity features into the holistic aero-structural optimization of long-span bridges.

**Keywords:** Aero-structural optimization, flutter, buffeting, nonlinear aeroelasticity, deck shape.

## 1 INTRODUCTION

The design of bridges has evolved in the last two centuries with the advances in structural analysis techniques and wind load modeling capabilities (Gimsing and Goergakis, 2012). The increasing length of the main spans of the long-span bridges built in the last century has turned the wind-induced loads into the governing design load. Advances in finite element modeling (FEM),

wind tunnel testing techniques, and the multi-model flutter and buffeting theories developed in the 1970s by Prof. Davenport and Prof. Scanlan have provided the basis for the current wind-resistant design methods used in the industry. While designers may adapt the design process for each project, the common feature is the sequential procedure involving multidisciplinary analyses carried out by different consultants. This prevents the implementation of iterative procedures limiting the design modifications to heuristic rules or experience-based design decisions. The process is described in Figure 1, as described by Duan and Chen, 1999, or in Honshu-Shikoku Bridge Authority, 1990, which can be simplified in the following steps: (1) the structural design of the bridge, using FEM and including dead and live loads; (2) assessment of the wind force coefficients of the bridge deck, typically obtained through sectional model tests; (3) aerodynamic stability verification combining FEM analyses and the aerodynamic loads; and (4) aeroelastic stability verification, based on wind tunnel tests of full bridge reduced-scale aeroelastic models or in multi-mode aeroelastic analyses and dynamic sectional model tests. The results of each step depend on the deck cross-section shape and bridge mechanical properties defined in the previous step, which indicates their interdependency.

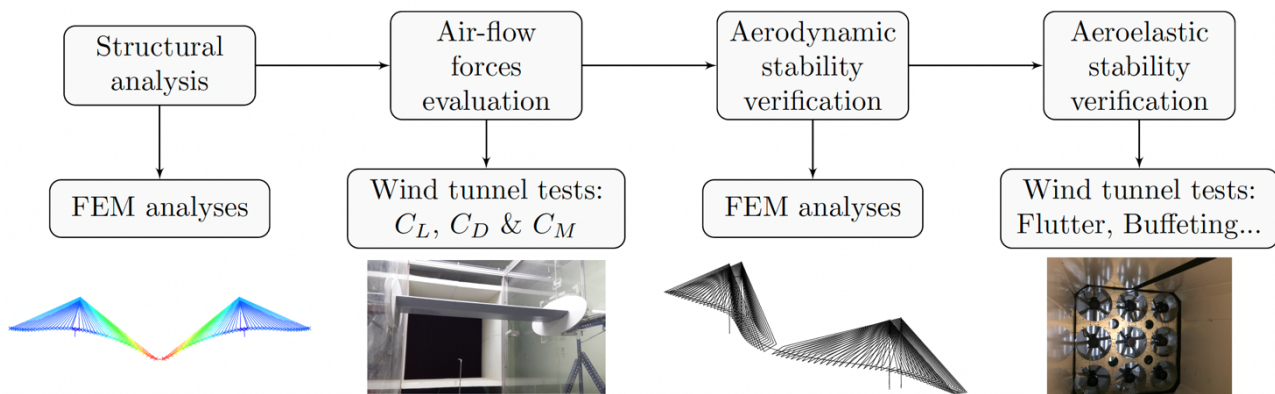


Figure 1: Flowchart of the classical approach for the wind-resistant analysis of long-span bridges.

These methods rely on wind tunnel testing, both sectional and full bridge aeroelastic models, and on the designer's experience to adopt design modifications if the performance is unacceptable. However, advances in CFD simulations (Selvam, 2022) to model wind-induced loads on bluff bodies, along with advances in metamodeling (Forrester et al. 2008) and optimization techniques (Hernández, 2010), have smoothed the path to the implementation of fully numerical methods for the design of bridges simultaneously considering the structural and aeroelastic performance.

## 2 AERO-STRUCTURAL DESIGN AND OPTIMIZATION OF LONG-SPAN BRIDGES

Aero-structural design strategies emerge as a fusion of wind-resistant and structural performance-based design to obtain designs that effectively withstand actions from both kind of loads. First aero-structural optimization methods were developed in the aerospace field for the design of aircraft wings (e.g., Martins et al. 2014). Recently, aero-structural design and optimization methods were developed for long-span bridges (Cid Montoya et al. 2018a, 2018b, 2022), efficiently combining CFD simulations, surrogate modeling, FEM analyses, optimization algorithms, and parallel computing in HPC clusters. The combination of CFD simulations with surrogate modeling permits an efficient exploration of the deck shape design domain that permits conducting the shape optimization of the deck in early design stages without depending on expensive wind tunnel tests.

While they are still required to validate the CFD simulations, the aforementioned interdependency of the design analysis is avoided (Cid Montoya et al. 2018a), facilitating the implementation of fully numerical design procedures led by optimization algorithms, as sketched in Figure 2.

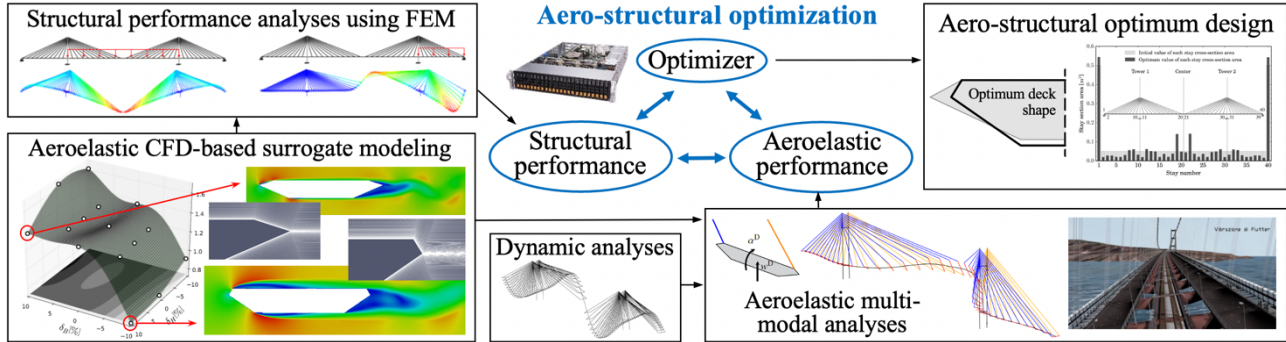


Figure 2: Conceptual flowchart of the aero-structural optimization framework for long-span bridges

## 2.1 Formulation of the aero-structural design optimization problem

The formulation must be defined to seek the definition of cost-effective, sustainable, safe, and resilient designs. An effective approach is the minimization of the material volume as an economic and sustainability indicator, which can be written in a general way as

$$\min F(\mathbf{S}_d, \mathbf{x}_s) = \min(V_D(\mathbf{S}_d, \mathbf{x}_s) + V_S(\mathbf{x}_s)) \quad (1)$$

Where  $V_D$  is the material volume of the deck,  $V_S$  stands for the material volume of the cable-supporting system,  $\mathbf{S}_d$  is a vector containing all shape design variables which control the aeroelastic and mechanic contribution of the deck to the global bridge performance,  $\mathbf{x}_s$  is a vector comprising all size design variables, including those of the deck, such as the deck plate thicknesses  $\mathbf{t}$ , and the cable-supporting system, such as stays cross-section areas  $\mathbf{A}$  and prestressing forces  $\mathbf{N}$ . The role of these variables in structural optimization problems is discussed in Atmaca et al., 2020, Cid et al., 2018, and Martins et al., 2020, among others. The problem is subject to structural design constraints that control the bridge's performance in its final (service) configuration and construction stages. The structural performance indicators comprise (1) vertical deck displacements; (2) horizontal displacements of the towers; (3) stress at the deck plates; and (4) axial stress in the stays, among others, under different combinations of dead and live loads. All these constraints can be written as

$$g_r^{Str}(\mathbf{x}) = \frac{R_r}{R_{r,max}} - 1 \leq 0, \quad (2)$$

where  $\mathbf{x}$  is the entire set of design variables,  $R_r$  is the structural response of the structural constraint  $r$ , and  $R_{r,max}$  is the maximum permitted value for each bridge response.

On the other hand, the design of wind-sensitive structures is conditioned by their aeroelastic performance under the specific wind condition of their location and along their entire life cycle, including construction, service, and long-term performance. The aeroelastic responses can be classified into two groups: (1) ultimate limit states that may jeopardize the integrity of the structure, such as aeroelastic (flutter) and aerostatic (divergence) instabilities; and (2) serviceability limit states

that affect the performance of the structure for their intended use, such as vortex-induced vibrations (VIV) or buffeting response (see ASCE 1992, JTGT 3360-01-2018, 2019). It must be noticed that the responses in the second group must also be limited to avoid fatigue issues. The thresholds for the ultimate limit states are formulated as the minimum critical wind velocity the structure must withstand  $R_{a,min}$ . On the other hand, the responses related to serviceability limit states are limited by maximum values  $R_{a,max}$  imposed in the design process. Hence, these constraints are formulated as shown below, depending on the kind of limitation (max or min):

$$g_{a,max}^{Aero}(\mathbf{x}) = \frac{R_a}{R_{a,max}} - 1 \leq 0, \quad \text{or} \quad g_{a,min}^{Aero}(\mathbf{x}) = \frac{R_{a,min}}{R_a} - 1 \leq 0, \quad (3)$$

The aeroelastic responses  $R_a$  must be computed using multi-mode analysis methods that provide the critical wind velocity in the case of the ultimate limit states (flutter and aerostatic instability) and the RMS and/or peaks of accelerations and/or displacements of multiple response points along the bridge deck and the three degree-of-freedom. The assessment of the aeroelastic responses along the optimization process requires the numerical estimation of the frequency-dependent fluid-structure interaction parameters (e.g., flutter derivatives and admittance functions). The self-excited and buffeting forces can be estimated based on the quasi-steady theory with reasonable accuracy for streamlined geometries at high reduced velocities by assuming frequency independency in modeling the frequency dependency by predefined functions, as it is typically done for the aerodynamic admittance (e.g., Davenport admittances) and small variations in the angle of attack. In this context, the aerodynamic information required is the set of force coefficients and their slopes. Hence, an aerodynamic surrogate model  $\mathcal{A}$  with the deck shape variables  $S_d$  an input and the aerodynamic information as output must be built to be used in the optimization process:

$$\mathcal{A}(S_d) = [C_D, C_L, C_M, C'_D, C'_L, C'_M], \quad (4)$$

Where  $C_D, C_L, C_M$  are the drag, lift and moment coefficients, respectively, and  $C'_D, C'_L, C'_M$  stand for their slopes with the angle of attack, which have a fundamental role in the definition of the most influential flutter derivatives and admittance functions.

## 2.2 Aero-structural optimum designs

The application of this methodology provides the aero-structural optimum design of the bridge, which is a design with the minimum economic cost that accomplishes all structural and aeroelastic design constraints by adopting the most advantageous configuration of the deck shape and deck plates thickness and cable-supporting system. This methodology was applied to single-box decks (Cid Montoya et al. 2018a, 2018b, 2022) and short-gap twin-box decks (Cid Montoya et al. 2021a, 2021b). A long-span cable-stayed bridge with a main span of 1316 m and two side spans of 540 m was chosen as an application example to test the capabilities of the methodology. The optimization algorithm could modify the design under different sets of requirements effectively. Figure 3 (a) compares the optimum deck shape obtained for 4 sets of buffeting constraints with increasing levels of demands (see Cid Montoya et al. 2022), which required the modification of the fairing angle to improve the aerodynamic performance of the deck in exchange of increasing the economic cost. Figure 3 (b) shows how the shape design variables, the width  $B$  and depth  $H$ , effectively change the RMS of buffeting vertical acceleration that controls the bridge performance.

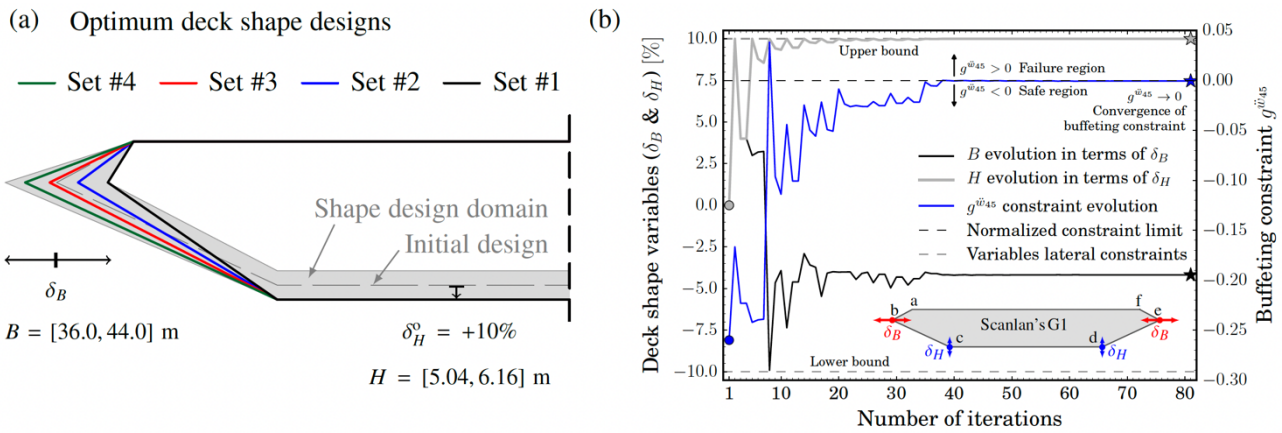


Figure 3: Optimum deck shape for different sets of buffeting design constraints.

### 3 LINEAR AND NONLINEAR AEROELASTICITY ANALYSIS METHODS

The results reported above were obtained by assessing the buffeting response of the bridge using the frequency domain analysis method and the QST. This methodology is accurate for streamlined geometries at high reduced velocities and subject to small variations of the instantaneous angle of attack. This methodology is the one typically used in the industry once the fluid-structure interaction parameters are available, as described above, and is based on the key contributions by Prof. Davenport for the buffeting response (Davenport, 1962a, 1962b) and Prof. Scanlan for the flutter instability analysis (Scanlan and Tomko, 1971) and later improvements by Jain et al., 1996, and Ge and Tanaka, 2000, Chen and Kareem, 2002, among others. Those methods recast the analytical expressions originally developed in the aerospace field by developing semiempirical strategies to model the aerodynamics of bluff geometries and are commonly defined as “linear” aeroelasticity methods.

However, full-scale monitoring measurements and wind tunnel test data showed the limitations of the so-called linear methods to model the wind-induced responses accurately (Diana et al., 1998). As highlighted by Chen and Kareem, 2003, “*Current linear aerodynamic force models have proven their utility for a number of practical applications, however, there are not suited for addressing completely the challenges posed by aerodynamic nonlinearities and turbulence effects.*” In the same paper, it is highlighted that “*Experimental studies have shown that the aerodynamic characteristics of many innovative bridge deck designs with attractive aerodynamic performance are very sensitive to the angle of incidence*”. The linear theory assumes small variations in the angle of attack around the static configuration. However, large instantaneous angles of attack can be found under multiple circumstances, such as large deck motion, large turbulent components, or a combination of both factors. These facts are commonly found in real constructions, as shown, for instance, in the full-scale measurements of the Humber Bridge, in the UK, reported by Boccione et al. 1992, which remarked that variations in the instantaneous angle of attack can be large. Similar conclusions were recently reported by Andersen et al. 2022 for the Gjemnessund Bridge, in Norway. Furthermore, Kareem and Wu 2013 remarked on the importance of including the hysteric behavior to model the nonlinear aeroelastic response properly. Results reported by Diana et al., 2010, showed that the aeroelastic forces using the linear theory can be underestimated, particularly in the case of the drag, which highlights “*the necessity of considering the nonlinear effects of large variations of the instantaneous angle of attack*”. These facts encouraged researchers to develop nonlinear analysis frameworks to overcome the limitations of linear methods. These methods include the corrected

quasi-steady theory model (Diana et al. 1993), band superposition model (Diana et al., 1995), hybrid nonlinear model (Chen and Kareem, 2001), the rheological models proposed by Diana et al. 2008, 2010, and further advanced in Diana and Omarini 2020, the ANN-based model by Wu and Kareem, 2011, and Volterra models by Carassale and Kareem 2010, among others. Interesting discussions on the performance of these methods can be found in Wu and Kareem, 2013, and Kavrakov and Morgenthal, 2017, among others.

#### 4 TOWARD NONLINEAR AERO-STRUCTURAL DESIGN OPTIMIZATION METHODS

Accurately modeling the bridge aeroelastic responses when nonlinearities control the response requires implementing time-domain methods in the aero-structural optimization problem. To address the relevant influence of the instantaneous angle of attack on the aeroelastic loads for each deck candidate design, a holistic aeroelastic surrogate model  $\mathcal{A}$  must provide the fluid-structure interaction parameters as a function of the deck shape  $S_d$ , angle of attack  $\alpha$  and reduced velocity  $U^*$  as:

$$\mathcal{A}(S_d, \alpha, U^*) = [A_i^*, H_i^*, P_i^*, \chi_{Du}^*, \chi_{Lu}^*, \chi_{Mu}^*, \chi_{Dw}^*, \chi_{Lw}^*, \chi_{Mw}^*], \quad (4)$$

where  $A_i^*, H_i^*, P_i^*$ , stand for the eighteen flutter derivatives, and  $\chi_{Du}^*, \chi_{Lu}^*, \chi_{Mu}^*, \chi_{Dw}^*, \chi_{Lw}^*, \chi_{Mw}^*$  are the admittance functions, both dependent on the deck shape, angle of attack and reduced velocity. Flutter derivatives and admittance functions should be obtained numerically (e.g., Mannini et al. 2016, Kavrakov et al. 2019) for a number of deck shape geometries as part of a sampling plan to train the surrogate. The first contributions in this direction were developed by Chen and Kareem, 2001, in interpolating the flutter derivatives using a rational function approximation. Recently, it was expanded by Barni et al. 2022 using a 2D rational function approximation to include the angle of attack. Different techniques for the same goal were used in the IABSE TG3.1. (Diana et al. 2022).

Another important nonlinear feature to include in the force model is the reproduction of the hysteresis loops in the response prediction schemes (Diana et al. 2010, Wu and Kareem, 2011). For instance, in the rheological models proposed by Diana and coworkers (Diana et al. 2008, 2010, Diana and Omarini 2020), the parameters used to adjust the expression of the aeroelastic forces must be expressed as a function of the deck shape by adopting surrogate strategies. This would lead to the following surrogate:

$$\mathcal{A}_{RM}(S_d, \alpha_0) = [r_1^0, k_1^0, r_1^1, k_1^1, r_1^1 + r_2^1, k_1^1 + k_2^1, m^2, r_1^2, k_1^2, r_1^2 + r_2^2, k_1^2 + k_2^2] \quad (5)$$

Where  $m^i, r_j^i$  and  $k_j^i$  are the parameters used to build the rheological model, where  $i$  stands for the order of the model. Similar strategies may be adopted for expanding the capabilities of the machine learning strategies proposed by Wu and Kareem, 2011, or alternative nonlinear models.

#### 5 CONCLUDING REMARKS

This paper summarizes the contributions of aero-structural design optimization methods in the wind-resistant design of long-span bridges, advancing the capabilities of current design approaches based on heuristic rules, linear aeroelastic methods, and wind tunnel testing. Implementing emulation techniques for the fluid-structure interaction parameters permits the development of fully numerical methods that the use of optimization methods can leverage. These methods can be further advanced

by improving the characterization of nonlinear features in aeroelastic load force modeling. Future directions to improve the design frameworks were discussed and will be developed and implemented into aero-structural design frameworks in forthcoming studies.

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