# Deterministic and probabilistic approaches for aeroelastic design optimization of long-span bridges
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**Santiago Hernandez**
hernandez@udc.es
Structural Mechanics Group, University of Coruña, School of Civil Engineering
A Coruña, Spain

**José A. Jurado**
jurado@udc.es
Structural Mechanics Group, University of Coruña, School of Civil Engineering
A Coruña, Spain

**Aitor Baldomir**
abaldomir@udc.es
Structural Mechanics Group, University of Coruña, School of Civil Engineering
A Coruña, Spain

**Miguel Cid Montoya**
miguel.cidmontoya@tamucc.edu
ASTRO Lab, Texas A&M University-Corpus Christi, School of Engineering
Corpus Christi, Texas, USA

**Ibuki Kusano**
Ibuki.kusano@iqs.url.edu
IQS, Ramon Llull University, School of Industrial Engineering
Barcelona, Spain

## ABSTRACT

The last decades have witnessed the construction of a number of long span bridges. Suspension bridges have reached main spans of more than 2000 m and cable stayed bridges of more than 1100 m. In addition to that, more challenging proposals are under steady. The main difficulty for these structures is to undergo the effects of earthquakes or aeroelastic phenomena and this paper is devoted to the latter class of loads, generated by wind flow. Giving the social relevance and cost of these constructions it is very important to use during the design the best technologies and numerical optimization methods are a powerful design tool. They have been applied since many years ago in other fields as aircraft or mechanical engineering but the idea of design optimization of long span bridges considering aeroelastic constraints is very recent. The optimization problem can be formulated as deterministic,-that means that all mechanical bridge properties and also the values assigned to loads, including wind related excitations, have fixed values-, or as probabilistic, which means that a level or uncertainty is included in the formulation, given the random nature of wind speed and the possible inaccuracies in the definition of bridge properties. This paper describes the formulations of aero-structural optimization of long span bridges considering flutter in both deterministic and probabilistic approaches. A long span cable stayed bridge and two suspension bridges, the Great Belt and the Messina bridges, are used as application examples of this methodology of design.

**Keywords:** Long span bridges, flutter speed, design optimization, Great Belt Bridge, Messina Bridge.

## 1 INTRODUCTION. THE RECENT ERA OF LONG SPAN BRIDGES

Bridges are constructions that interest people of all ages and types from the "man of the street" to top scientist. Among bridge typologies, very long span bridges are complex and at the same time delicate structures that require designers with top technical capabilities and experience. They transform radically the geometry of countries and even continents, in the last decades long span suspension bridges were used to cross the Great Belt in Denmark and to connect the Japanese islands of Honshu and Shikoku; one of them, the Akashi Bridge has a main span of 1991 m length. Chinese suspension bridges as the Xihoumen, Nansha and Yansigang have span lengths from 1650 m to 1700 m. Three outstanding long span bridges have been built in Turkey since 2016: the Yavuz Sultan Selim Bridge, the Osman Gazi Bridge and the Canakkale Bridge, that with a main span of 2025 m is the world longest span. The design of a suspension bridge over the Messina strait in Italy is completed and a suspension bridge with two main spans that more than one kilometre each one is under construction in Chile over the Chacao strait. In Spain, the ERA 2000 project aimed to cross two straits in the Northwestern Atlantic coast contains two suspension bridges with more than two kilometres of main span length (Hernandez, 2001). In the case of cable stayed bridges there are many examples of bridges with more than 800 m of central span and the Stonecutters Bridge, the Sutong Bridge, the Hutong Bridge in China and the Russky Bridge in Russia have span lengths ranking from 1018 to 1104 m. Several studies considers cable stayed bridges of about 1.5 km (Ge, 2016).

The advances in materials, construction procedures and design methodologies allow to consider than the trend of increasing long span in suspension and cable stayed bridges will continue in the future. These bridges support highway traffic and high-speed trains but the main concern in their designs is not the load associated with cars or trains but the effects generated by natural forces as wind flow; and they need to be designed carefully to guarantee an efficient performance.

## 2 FORMULATION OF FLUTTER ANALYSIS MAIN WIND INDUCED PHENOMENA

Flutter is an instability phenomenon that leads to bridge collapse. It is studied by defining self-excited forces $L^{se}$, $M^{se}$, $D^{se}$ (Jain et al., 1996) as presented in figure 1. They are expressed as

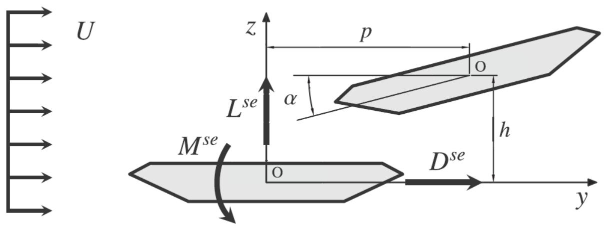



Figure 1. Sign of self-excited aerodynamic forces

$$D^{se} = \frac{1}{2}\rho U^2 B\left( KP_1^* \frac{\dot{p}}{U} + KP_2^* \frac{B\dot{\alpha}}{U} K^2 P_3^* \alpha + K^2 P_4^* \frac{p}{B} + KP_5^* \frac{\dot{h}}{U} + K^2 P_6^* \frac{h}{B} \right) \quad (1a)$$

$$L^{se} = \frac{1}{2}\rho U^2 B\left( K H_1^* \frac{\dot{h}}{U} + K H_2^* \frac{B\dot{\alpha}}{U} K^2 H_3^* \alpha + K^2 H_4^* \frac{h}{B} + K H_5^* \frac{\dot{p}}{U} + K^2 H_6^* \frac{p}{B} \right) \quad (1b)$$

$$M^{se} = \frac{1}{2}\rho U^2 B\left( K A_1^* \frac{\dot{h}}{U} + K A_2^* \frac{B\dot{\alpha}}{U} K^2 A_3^* \alpha + K^2 A_4^* \frac{h}{B} + K A_5^* \frac{\dot{p}}{U} + K^2 A_6^* \frac{p}{B} \right) \quad (1c)$$

Where $L^{se}$, $M^{se}$, $D^{se}$ are the unit vertical, horizontal and moment components, $h$, $p$, are the vertical and horizontal displacements, $\alpha$ the rotation angle, $\dot{h},\dot{p},\dot{\alpha}$ corresponds to their respective velocities, $H_i^*,A_i^*,P_i^*$ $(i=1,...6)$ are the set of flutter derivatives and $K=Bw/U$ is the reduced frequency. A simplification of this scheme is the so called quasi-steady load model (Scanlan, 1988). This formulation uses a reduced number of flutter derivatives. With expression (1)) the dynamic equilibrium of the bridge can be written as (2) and the solution of the eigenvalue problem provides the flutter wind speed $U_f$.

$$\mathbf{M\ddot{u}}+\left(\mathbf{C}-\mathbf{C}^{se}\right)\mathbf{\dot{u}}+\left(\mathbf{K}-\mathbf{K}^{se}\right)\mathbf{u}=\mathbf{0} \tag{2}$$

# 3 DETERMINISTIC OPTIMIZATION OF SHAPE DECK AND CABLE CROSS-SECTION OF LONG-SPAN CABLE-STAYED BRIDGES SUBJECT TO STRUCTURAL AND AEROELASTIC CONSTRAINTS

## 3.1 Formulation

The modern formulation of structural optimization is defined (Schmit,1960, Hernández, 2010) as a non linear constrained optimization problem in which the purpose is to identify the values of a set of design variables **X** than produce the best value of a function $F(\mathbf{x})$ coined objective function while accomplishing a number of conditions, also labelled constraints $g_j(\mathrm{X})\leq 0 \quad (j=1,...n)$. Since then this technique has been progressively introduced in several fields as aerospace, mechanical or civil engineering. In the case of long span bridges the first examples of application of numerical optimization including constraints related to aeroelastic phenomena, namely flutter, started some years ago (Jurado et al.,2008) and (Nieto et al., 2009). These studies optimized the thicknesses of the plates of the deck cross-section but maintained deck shape. The first attempt to modify the geometry of the deck in the optimization procedure is more recent (Cid Montoya et al., 2018) and carried out the optimization of deck shape and areas of the cables in long-span cable-stayed bridges. It must be considered that the modification of deck geometry alter the value of the aerodynamic properties as aerodynamic coefficients or flutter derivatives. Optimization techniques are iterative processes in which the design values change at each iteration and so do the aerodynamic properties of the deck. In that regard, the mean to obtain these magnitudes cannot be wind tunnel test and a fully numerical procedure is needed. An efficient approach is to carry out a number of CFD simulations with values of the design variables that map appropriately their range of variation. With the results obtained of each aerodynamic property a surrogate model can be generated (Forrester et al. 2008); this tool will provide the input for each values of the set of the design variables along the iterations of the optimization process. This fully numerical optimization methodology is presented more extensively in the next section applied to a long span cable-stayed bridge.

## 3.2 Aero-structural optimization of deck shape and cable area of a long span cable stayed bridge under structural and flutter constraints

The mentioned formulation has been applied to the long span cable stayed bridge presented in figure 1. The cross-section of the deck is the single box G1 (Scanlan and Tomko,1971) presented in figure 2. Magnitudes B an H are the witdh and depth of the deck and represent the shape design variables, they are allowed to vary in the range of $\pm 10\%$ with regards to the initial value. Top, bottom and side plates have the same thickness $t$ than can vary from 0.01 an to 0.03 cm. The rest of the design variables are the cross areas of the cable **A** and their prestressing forces **N**.

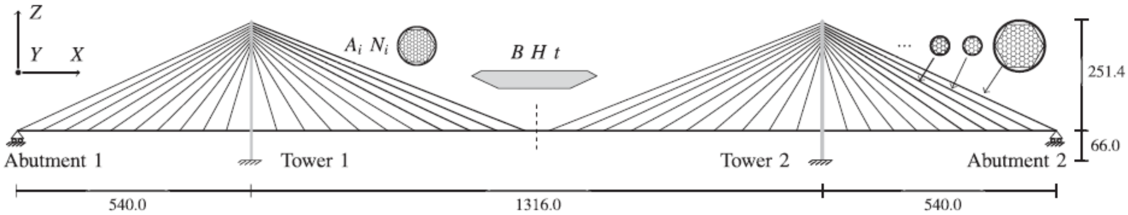
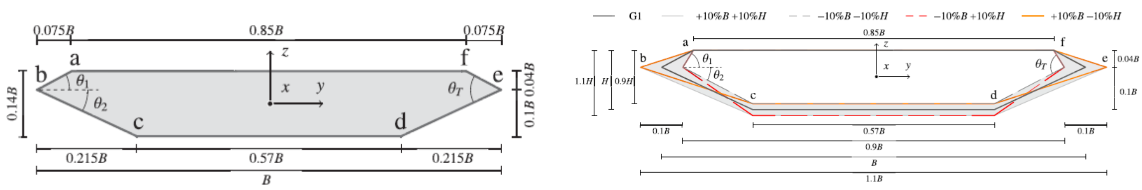



Figure 1: Long span cable stayed bridges




a) Shape geometry b) Variation of shape design variables

Figure 2: Cross-section of the cable stayed bridge

The static loads considered in the process are self-weight and live loads; the flutter speed is calculated according to the quasi-steady theory mentioned previously. The objective function of the optimization is the addition of the volume of the deck and the cables as written in expression (3), where $A_x$ is the area per unit length of the deck, $L_D$ the bridge length and $A_i$, $L_{S,i}$ the area and length of the i-esime cable. Structural constraints correspond to upper values of stresses and displacements of deck and tower top and can be written as (4). Finally, the flutter constraint is presented in (5).

$$minF(B,H,t,\mathbf{A},\mathbf{N}) = A_x\left(B,H,t\right)L_D + 2\sum_{i=1}^{40} A_x\, L_{S,i} \tag{3}$$

$$g_r^{Str}\left(\mathbf{x}\right) = \frac{R_r}{R_{r,max}} - 1 \leq 0, \qquad r = 1,\ldots 1104 \tag{4}$$

$$g^{U_f}\left(\mathbf{x}\right) = \frac{U_{f,min}}{U_f} - 1 \leq 0, \tag{5}$$

The flowchart of the process appears in figure 3, showing the CFD simulations needed for the surrogate models and the rest of multidisciplinary analysis involved in the optimization problem.

The optimization process was performed for different values of flutter speed, namely [115,120,125,130] m/s, and Figure 4 shows the optimum shape of the deck for each case.

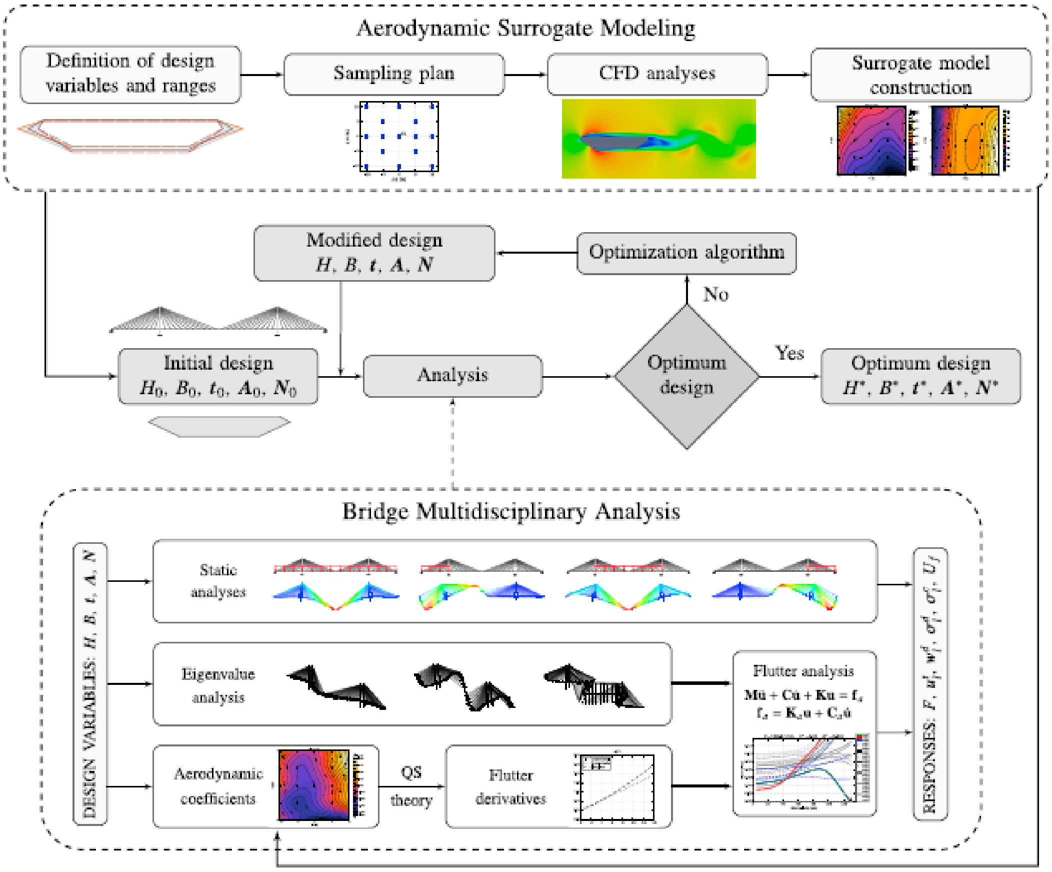



Figure 3: Flowchart of the aero-structural optimization.

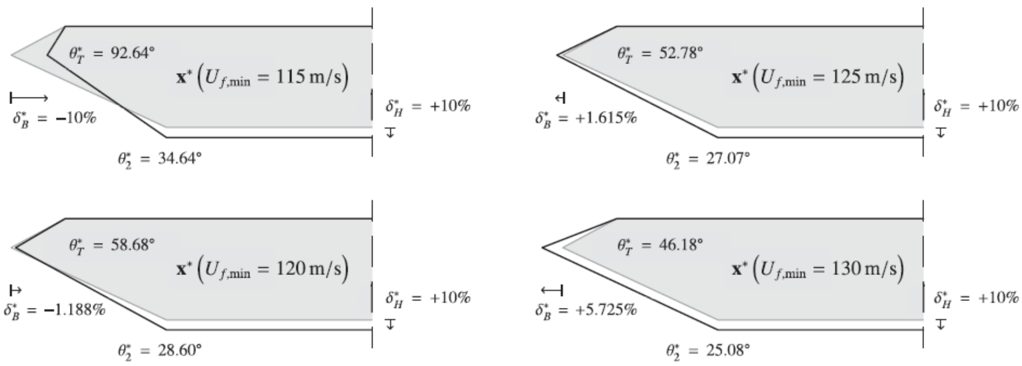



Figure 4: Optimum shape for a set of flutter speeds $U_f$ = [115,120,125,130] m/s

# 4 PROBABILISTIC OPTIMIZATION OF DECK SHAPE OF A LONG SPAN BRIDGE SUBJECT TO FLUTTER AND MECHANICAL CONSTRAINTS

## 4.1 Formulation of probabilistic optimization

Probabilistic optimization is usually coined as RBDO (Reliability Based Design Optimization) and commits on considering that some properties of the problem, for instant loads values and structural properties are random variables. In that cases the responses of the structure related to the random variables do not have a fixed value, on the contrary they have a random distribution and therefore if they are included in the set of constraints of the optimization process there is always a probability of being violated, in other words there is a probably of failure. Thus, in RBDO the designer has to define the probability of failure $P_f$ accepted for the constraint or the reliability index $\beta$ that is related with $P_f$ by the expression $P_f = \Phi(-\beta)$ where $\Phi$ is the cumulative distribution function of a Gaussian function. Therefore, the RBDO formulation can be written as (Kusano et al., 2014).

$$min\, F(\mathbf{d}) \tag{6}$$

$$P\left[G_i(\mathbf{d},\mathbf{u}) \le 0\right] \le P_{fi}^T \qquad i = 1,\ldots m \tag{7}$$

$$g_j(\mathbf{d}) \le 0 \tag{8}$$

Where **d** is a vector of design variables, **u** a vector of the normalized random variables, $P$ a probability operator, $P_{fi}^T$ a allowable probability of failure, $G_i$ each of the limit state functions, $g_j$ the deterministic constraints, $m$ the number of limit state function and $M$ the total number of constraints. There are several techniques to proceed in RBDO, in this research the Reliability Index Approach (RIA) (Nikolaidis and Burdisso, 1988), was used in the application example.

### 4.2 RBDO of deck shape of a suspension bridge under flutter and structural constraints

The RBDO formulation has been applied to a long-span suspension bridge with the geometry of the Great Belt Bridge as presented in figure 5 (Kusano et al. 2020).

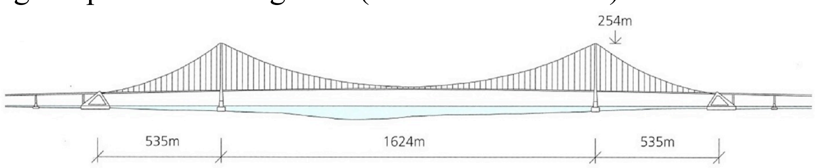



Figure 5: Size view of the Great Belt Bridge

The cross section of the of the deck appears in Figure 6 and the shape design variables shape are composed by the width δB, depth δH and the thickness $d_i$ (i=1,..,4) of the top, lateral and bottom plates as appears in figure 7. The random variables were composed by aerodynamic coefficients $C_L$, $C_M$, $C_D$ their slppes at zero angle of attack , $C'_{L,0}, C'_{M,0}$ and $C'_{D,0}$ and wind speed that had a Gumbel distribution expressed by (9) with $\mu = 41{,}60$ and $\lambda = 2425$.

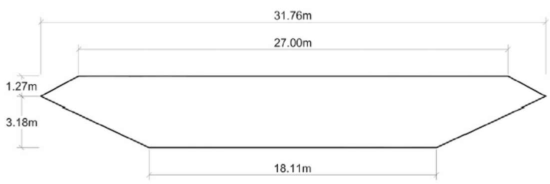



Figure 6: Shape of deck

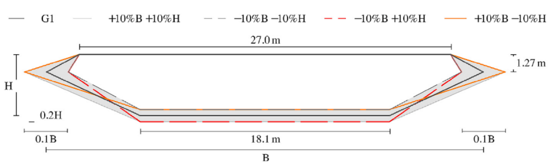



Figure 7: Shape design variables

$$f_G(x_i) = \frac{1}{\lambda}\exp\left(-\frac{x_i - \mu}{\lambda}\right)\cdot\exp\left[-\exp\left(-\frac{x_i - \mu}{\lambda}\right)\right] \tag{9}$$

Aerodynamics coefficients were obtained by surrogate methods using a number of CFD simulations that led to a normal distribution with a coefficient of variation (CV) of 0.2, the standard deviation of the shapes of the aerodynamic coefficients were calculated as

$$\sigma'_{c_i} = \frac{\sqrt{\left(\sigma_{c_i 0^o}\right)^2 + \left(\sigma_{c_i 2^o}\right)^2 + 2\rho \cdot \sigma_{c_i 0^o} \cdot \sigma_{c_i 2^o}}}{\Delta\alpha} \tag{10}$$

where $\sigma'_{c_i}$ is the standard deviation of slope of a force coefficient, $\sigma_{ci0^o}$ and $\sigma_{ci2^o}$ are the standard deviations of the coefficient at 0° and 2° degrees of angle of attack respectively, $\rho$ is the correlation, and $\Delta\alpha$ is the change in angle in radians. Since force coefficients at 0° and 2° are highly correlated, $\rho = 1$ was assumed as the most conservative case. The $\Delta\alpha$ value of 2° is considered in the calculations.

The flutter derivatives were evaluated using the quasi-steady theory described in a previous section so, the formulation of RBDO was

$$\text{Minimize: Girder volume } (\delta H, \delta B, d_1, d_2, d_3, d_4) \tag{11}$$

$$P\left[V_f(\mathbf{x}) - x_w \leq 0\right] \leq P_f \tag{12}$$

$$g_2: \text{-}10\% \leq \delta H \leq 10\% \tag{13}$$

$$g_3: \text{-}10\% \leq \delta B \leq 10\% \tag{14}$$

$$g_4: 7 \text{ mm} \leq d_j \leq 25 \text{ mm} \quad \text{j=1,2,3,4} \tag{15}$$

$$g_5: \sigma_c = 565 \text{ MPa} \tag{16}$$

$$g_6: \frac{z_d}{z_{max}} - 1 \leq 0 \tag{17}$$

Where

$$z_{max} = L/500; \; L = 1624 \text{ m}$$

The shape design variables $\delta H$ and $\delta B$ range from -10% to +10% of the original dimension aiming to avoid infeasible shapes for the box girder. The $g_6$ limits the maximum vertical displacement of the bridge deck under the traffic overload case based on BS 5400 (British Standards Institution, 2000), in which a full load of 2.4 kN/m$^2$ was applied to the two of the six lanes while 1/3 of the load was applied to the other lanes. The constraint $g_5$ is used to assign the main cable area whenever the deck weight changes so that the main cable stress is always at 565 MPa. The results for different values of the reliability index $\beta^T$ are presented in table 1.

Table 1. RBDO results for different values of $B^T$

| $\beta^T$ | $V_f$ | $\Delta H$ | $\Delta B$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | Obj. func | % variation obj.func. |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 75.09 | 10.00 | 4.80 | 8.40 | 8.44 | 7.33 | 8.49 | 2409.19 | -13.33 |
| 8 | 82.10 | 9.97 | 3.83 | 9.52 | 10.64 | 9.46 | 10.63 | 2718.78 | -2.19 |
| 9 | 86.95 | 9.99 | 0.89 | 12.50 | 11.86 | 11.23 | 12.57 | 3058.50 | 10.03 |
| 10 | 92.70 | 7.15 | 0.46 | 17.16 | 16.04 | 11.14 | 13.54 | 3628.01 | 30.52 |

## 5 CONCLUSIONS

According to the information presented in this paper, it can be concluded that the methodology of aero-structural optimization is a mature discipline that can be applied to real structures in civil engineering on long-span bridges. It involves the use of several disciplines as CFD simulations, surrogate models, and nonlinear numerical optimization algorithms.

It can deal not only with deterministic magnitudes but also with properties of random nature in the loads and the properties of the bridge. This approach is closer to the real situation of the design process of the structures.

The application examples of cable-stayed and two suspension bridges reproduce with a quite significant degree of accuracy the context of the bridge's design under flutter and stress, and displacement considerations. More research is being carried out to enhance this methodology and include in the formulation more aeroelastic phenomena.

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